Privacy-Preserving Distributed Cooperative Command Governor Schemes

Ayman El Qemmah, Alessandro Casavola and Francesco Tedesco

***Abstract*— A novel distributed Command Governor (CG) scheme, relying on cooperative distributed optimization, is presented for multi-agent networked systems subject to local and coupling constraints. Unlike existing non-cooperative distributed CG schemes, the proposed approach leverages a distributed optimization framework based on relaxation techniques and duality theory to enable agents to cooperatively contribute individually to the minimization of a global performance index. Furthermore, agents exchange only a vector that encodes the resource utilization of other agents, preserving privacy by avoiding the exchange of information about objective functions, constraints, or admissible commands. The effectiveness of the proposed approach is validated through an illustrative example involving vehicles operating in a 2D space that are subject to connectivity keeping coupling constraints.**

***Index Terms* — Multi-Agent Systems, Cooperative Command Governor, Distributed Optimization, Constrained Control, Distributed Supervision Scheme.**

I. Introduction

Command Governors (CG) [1], [2], [4] are effective tools for enforcing constraints on pre-compensated systems, particularly in industrial applications where peripheral constraint handling is required. Early CG formulations employed a centralized architecture, where a single supervisory unit determined the solution of the optimization problem for all systems. To address the limitations posed by the use of a centralized unit in complex networked systems, recent research has focused on distributed approaches. These methods involve distributing the optimal control problem among network agents, each solving a local optimization problem.

Existing distributed CG strategies, such as [6], [7], [8], [9], [10], are mainly based on non-cooperative game-theoretical approaches that can lead to deadlock situations and undesirable Nash equilibria [6]. More in detail, in non-cooperative CG strategies each agent optimizes a local version of the global centralized cost index without any agreement that enforce the cooperation with the other agents. This can lead to competitive and selfish behaviors and suboptimal solutions. In order to avoid "egoistic behavior," the interest here is moving in the class of cooperative methods. These methods, which are now receiving a growing interest, see e.g. [11], aim to "cooperatively optimize" a global objective function, splitted in a combination of local objective functions, while relying solely on local data. Fortunately, the optimization set-up of the centralized CG is amenable to distributed computation, since it involves minimizing the sum of local cost functions, each of which depends on local variables only, subject to local and coupling constraints involving all local and global decision variables.

The main approach used here involves the use of Lagrangian dual-decomposition and dual methods. This leads to distributed optimization algorithms when the problem is "separable," i.e. problems where local-objective functions and constraints can be decomposed based on the components of the decision vector. This approach has been extensively used to design cross-layer resource-allocation mechanisms [14], [15], [16], [17]. Further distributed optimization algorithms for constraint-coupled problems have been explored in [18], [19], [20], [21] while recent works based on techniques like consensus-based primal-dual perturbation [22], dual decomposition [23], [24], saddle-point subgradient [25], and cutting-plane consensus [26] have been proposed, but challenges remain, such as primal feasibility recovery and the need for agents to agree on the complete solution vector.

In [12], a distributed iterative CG method that induces cooperation was proposed, based on a distributed penalization method [13], where the local performance index of each agent also incorporates terms related to the collective goal. However, that scheme is not able to obtain in a distributed way the same solution of the centralized CG approach. Moreover, because the local decision variables are made available, the privacy of the agents might result compromised. In order to enhance the resilience, in the proposed approach the privacy requirement is prioritized by preventing agents from communicating each other local decision variables, costs, or constraints.

Based on the above considerations, in this paper, we propose a novel distributed cooperative CG supervision scheme by exploiting the distributed optimization method proposed in [27]. The Relaxation and Successive Distributed Decomposition (RSDD) algorithm of [27], characterized by its simplicity and local optimization structure, leverages relaxation techniques and duality theory to ensure convergence to optimal centralized cost and primal recovery, without relying on averaging mechanisms, and, notably, ensuring privacy preservation through the exchange of only auxiliary variables.

However, because constraints fulfilment is reached only asymptotically in iterative cooperative schemes, our interest here is also to slightly tighten the constraints to achieve in a finite number of iterations a solution able to enforce the original constraints, with such a number of iterations being determined by a stopping criterion. This inevitable results in a feasible solution that is a near-optimal solution of the

A. El Qemmah, A. Casavola and F. Tedesco are with Dipartimento di Elettronica, Informatica e Sistemistica, Università degli Studi della Calabria, Via Pietro Bucci, Cubo 42-c, Rende (CS), 87036, Italy, {elqemmah.ay,a.casavola,ftedesco}@dimes.unical.it

original centralized CG problem.

The paper is organized as follows. Section 2 presents the basic notation and recalls the general formulation of the Command Governor approach. Section 3 formally states the problem and Section 4 briefly introduces the theory behind the RSDD algorithm and presents the distributed cooperative CG solution. Section 5 describes the properties of the solution while Section 6 presents an illustrative example of vehicles operating in a 2D space subject to connectivity keeping constraints to assess the effectiveness of the proposed approach. Finally, in Section 7 we summarize the main results of this work and some conclusions end the paper.

II. Preliminaries

Definition 2.1: **(Pontryagin-Minkowski Difference)**: Given two sets $\mathcal{D}_1, \mathcal{D}_2 \in \mathbb{R}^n$, the Pontryagin-Minkowski Difference is defined as

$$\mathcal{D}_1 \sim \mathcal{D}_2 := \{d_1 | d_1 + d_2 \in \mathcal{D}_1, \forall\, d_2 \in \mathcal{D}_2\}.$$

Definition 2.2: (**Graph**): A graph is an ordered pair $\Gamma(\mathcal{A}, \mathcal{E})$ such that

- $\mathcal{A}$ is the set of nodes;
- $\mathcal{E} \subseteq \mathcal{A} \times \mathcal{A}$ is a subset of pairs of $\mathcal{A}$ known as the set of edges connecting two nodes.

Definition 2.3: (**Neighborhood of the i-th node**): The neighborhood $\mathcal{N}_i$ of the i-th node in $\Gamma(\mathcal{A}, \mathcal{E})$ consists of all its adjacent nodes i.e.

$$\mathcal{N}_i = \{j \in \mathcal{A} : (i, j) \in \mathcal{E}\}. \quad (1)$$

Definition 2.4: (**Local states of neighbors of the i-th node**): The sets of all states x_j related to the i-th agent can be characterized as follows

$$[x]_i = \{\text{All subvectors } x_j \text{ of } x \text{ such that } j \in \mathcal{N}_i\} \quad (2)$$

A. The Command Governor

In this section, the CG strategy is briefly reviewed. To this end, consider $N \in \mathbb{N}$ linear time-invariant closed-loop discrete-time dynamical systems, where each i-th subsystem is modeled as

$$\Sigma_i : \begin{cases} x_i(t+1) &= \Phi_{ii} x_i(t) + G_i g_i(t) + \sum\limits_{j \in \mathcal{A}-\{i\}} \Phi_{ij} x_j(t), \\ y_i(t) &= H_i^y x_i(t), \\ c_i(t) &= H_i^c x(t) + L_i g(t), \end{cases} \quad (3)$$

where, for all $i \in \{1, \ldots, N\}$, $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector (which includes the controller states under dynamic regulation) at time $t \in \mathbb{Z}_+$. Vector $g_i(t) \in \mathbb{R}^{m_i}$, that is the manipulable command vector, in the absence of constraints (and no CG), would coincide with the desired reference $r_i(t) \in \mathbb{R}^{m_i}$, assuming each system is regulated by a local controller that ensures stability and satisfactory closed-loop performance. The vector $y_i(t) \in \mathbb{R}^{m_i}$ is the output vector related to the tracking performance ($y_i(t) \approx r_i(t)$). Furthermore, $c_i(t) \in \mathbb{R}^{n_i^c}$ is the constrained vector that collects all constrained local variables, i.e. $c_i(t) \in \mathcal{C}^i, \ \forall t \in \mathbb{Z}_+, i = \{1, \ldots, N\}$, with $\mathcal{C}^i$ being convex and compact polytopic sets.

From a global point of view, the overall system Σ arising by the composition of the above N subsystems (3) can be described as

$$\Sigma : \begin{cases} x(t+1) &= \Phi x(t) + G g(t), \\ y(t) &= H^y x(t), \\ c(t) &= H^c x(t) + L g(t), \end{cases} \quad (4)$$

where, to describe global (coupling) constraints among states of different subsystems, the vector c_i in (3) is allowed to depend on the aggregate state and manipulable commands vectors $x = [x_1^T, \ldots, x_N^T]^T \in \mathbb{R}^n$, with $n = \sum_{i=1}^N n_i$, and $g = [g_1^T, \ldots, g_N^T]^T \in \mathbb{R}^m$, with $m = \sum_{i=1}^N m_i$. Moreover, the other relevant aggregate vectors r, $y \in \mathbb{R}^m$ and $c \in \mathbb{R}^{n^c}$, with $n^c = \sum_{i=1}^N n_i^c$, are defined analogously. $\Phi = [\Phi_{ij}]_{i,j=1,\ldots,N}$, $G = diag(G_1, ..., G_N)$, $H^y = diag(H_1^y, ..., H_N^y)$, $H^c = [(H_1^c)^T, ..., (H_N^c)^T]^T$ and $L = [(L_1)^T, ..., (L_N)^T]^T$.

Hereafter, it is further assumed that Φ *is a strictly Schur matrix*. From a distributed perspective, the above assumption implies that the local regulators to each subsystem (3) are capable to asymptotically stabilize the overall system (4).

Then, the CG design problem involves determining, at each time step t and for each agent $i \in \mathcal{A}$, a suitable command $g_i(t)$ that best approximates the reference $r_i(t)$ while ensuring no constraint violations, i.e. $c_i(t) \in \mathcal{C}^i, \forall t \in \mathbb{Z}_+, i \in \{1, \ldots, N\}$, under the application of a suitably computed sequence of feasible $\{g_i(t)\}_{t=0}^{\infty}$. In [1], a centralized solution to the previously stated CG design problem was proposed. In this approach, the CG computed command $g(t)$ is determined at each time instant t as a function of the current reference $r(t)$ and measured state $x(t)$, ensuring that $g(t)$ is the best feasible approximation of $r(t)$ compatible with the overall constraints representation

$$c(t) \in \mathcal{C} := \mathcal{C}^1 \times \mathcal{C}^2 \times \ldots \times \mathcal{C}^N, \forall t \in \mathbb{Z}_+ . \quad (5)$$

To introduce the standard centralized solution, it is useful to define the steady-state of the constrained variable c as $c_g := H^c(I_n - \Phi)^{-1} G g + L g$, and the virtual c-evolutions when a constant command sequence $g(\kappa) \equiv g$ is applied to Σ, along the virtual time κ starting at time $\kappa = 0$, starting from the initial aggregate state $x(t)$ as

$$c(\kappa, x(t), g) := H^c \Phi^\kappa x(t) + \left(\sum_{i=0}^{\kappa-1} \Phi^{k-i-1} G g \right) + L g. \quad (6)$$

As demonstrated in [3], instead of ensuring constraints satisfaction for all $\kappa \geq 0$, it suffices to introduce steady-state constraints with a margin $\delta > 0$ and evaluate virtual predictions $c(\kappa, x(t), g)$ within a specific *admissibility index* κ_0. The centralized CG action is then determined by solving the following constrained optimization problem

$$g(t) = \arg \min_{g \in \mathcal{V}(x(t))} \|g - r(t)\|_\Psi^2, \quad (7)$$

where $\Psi = \Psi^T > 0$ and $\mathcal{V}(x)$ is defined as

$$\mathcal{V}(x) := \{g \in \mathcal{W}_\delta : c(\kappa, x, g) \in \mathcal{C}, \ \kappa = 0, \ldots, \kappa_0\}, \quad (8)$$

where $\mathcal{W}_\delta$ represents the set of all constant commands whose corresponding steady-state of constrained variable c satisfies the constraints with margin δ, i.e.

$$\mathcal{W}_\delta := \{g \in \mathbb{R}^m : \ c_g \in \mathcal{C} \sim \mathcal{B}_\delta\}, \quad (9)$$

with $\mathcal{B}_\delta := \{x \in \mathbb{R}^n : ||x|| \leq \delta\}$ a generic ball in an Euclidean space $\mathbb{R}^n$. Due to the polytopic nature of $\mathcal{C}^i$, $\mathcal{V}(x)$ can be characterized by a finite number z of inequalities in $\mathbb{R}^m$ ([5]) that can be expressed in matrix form

$$\mathcal{V}(x) := \{g \in \mathbb{R}^m : Rg + \tilde{R}x - v \leq 0\}, \tag{10}$$

with matrices R, $\tilde{R}$ and v properly evaluated. By denoting $h(g, x) := Rg + \tilde{R}x - v$, h being an affine function $h : \mathbb{R}^{m+n} \to \mathbb{R}^z$, an equivalent form for (7) is

$$\begin{aligned} &\min_{g \in \mathbb{R}^m} \ \|g - r(t)\|^2_\Psi \\ &\text{s.t} \quad h(g, x(t)) \leq 0. \end{aligned} \tag{11}$$

The solution for the above problem (11) represents the *best* feasible approximation, of $r(t)$ which, if constantly applied from t onwards never produces constraints violation. It is noteworthy that, if $r(t) \notin \mathcal{V}(x(t))$, the solution of (11) is significantly influenced by the shape of the weighting matrix Ψ, which reflects the preferences of the designer. A particular case of interest here, contemplates the use of a block-diagonal matrix Ψ in (11), that is

$$\begin{aligned} &\Psi := diag(w_1\Psi_1, \ldots, w_N\Psi_N), \\ &\Psi_i \in \mathbb{R}^{m_i \times m_i},\ \Psi_i = \Psi_i^T > 0_{m_i}, w_i \in \mathbb{R}_{>0},\ \forall i \in \mathcal{A}. \end{aligned} \tag{12}$$

III. Problem Formulation

In scenarios involving multiple agents, the enforcement of coupling constraints is often required. These constraints arise in various applications, such as resource sharing among multiple systems or maintaining connectivity between mobile robots. In order to explicitly express the contribution of the i-th agent, the coupling constraint can be expressed in the form in the form $\sum_{i=1}^{N} h_i^c(g_i, x) \leq 0$, where each $h_i^c : \mathbb{R}^{m_i+n} \to \mathbb{R}^s$ is assumed to be convex. Please notice that inequality signs are meant component-wise for vectors. Considering (12) and $\bar{\Psi}_i = w_i\Psi_i$, problem (11) can be recast as

$$\begin{aligned} \min_{g_1,\ldots,g_N} \quad & \sum_{i=1}^{N} f_i(g_i, r_i(t)), \\ \text{s.t.} \quad & g_i \in \mathcal{V}_i^l(x_i(t)), \qquad i \in \{1, \ldots, N\} \\ & \sum_{i=1}^{N} h_i^c(g_i, x_i(t), [x]_i(t)) \leq 0, \end{aligned} \tag{13}$$

where, for all $i \in \{1, \ldots, N\}$, $f_i(g_i, r_i(t)) = \|g_i - r_i(t)\|^2_{\bar{\Psi}_i}$, the set $\mathcal{V}(x)$ has been split, $\mathcal{V}_i^l(\cdot) \subseteq \mathbb{R}^{m_i}$ (where the superscript l stands for "local") is a non-empty, compact, convex set, and $h_i^c : \mathbb{R}^{m_i+\check{n}_i} \to \mathbb{R}^s$, with s that depends on κ_0 and $\check{n}_i = n_i + |\mathcal{N}_i|$, where $|\mathcal{N}_i|$ is the cardinality of $\mathcal{N}_i$.

As previously discussed, a centralized implementation of the optimization problem (13) requires a central computational facility with access to all system variables. In contrast, this paper focuses on designing a distributed solution that involves N computational nodes, each with limited information about the overall system, where each node determines locally an admissible command $g_i(t)$ at each time step t.

In this context, we consider a network of N agents that communicate via a *connected*, *undirected* graph $\mathcal{G} = (\mathcal{A}, \mathcal{E})$, where $\mathcal{A} = \{1, \ldots, N\}$. Each agent i is assumed to have a limited information scope, only regarding $j \in \mathcal{N}_i$. Please notice that, for agent i, in (8) $\exists$ at least a line r_p^T in R or a line r_x^T in $\tilde{R}$ such that ($r_p^i \neq 0$ and $r_p^j \neq 0$), ($r_x^i \neq 0$ and $r_x^j \neq 0$) respectively, for all $j \in \mathcal{N}_i$.

Assumption *1:* We assume that each agent $i \in \mathcal{A}$, has knowledge only about its local constraint function $\mathcal{V}_i^l(\cdot)$, its local utility function f_i, and its own contribution h_i^c to the coupling. To preserve privacy, agents are required not to share information about admissible commands, costs, or constraints.

The problem to be solved in the distributed setting can be stated as follows.

Problem *1: Locally determine , at each time step t, for all $i \in \mathcal{A}$, a suitable command signal $g_i(t)$ such that the resulting aggregate vector $g(t)$ is the optimal solution for problem* (13).

IV. Distributed Optimization Based on a Relaxation and Duality Approach

In this section, the main goal is to formally state a distributed algorithm from the perspective of node i to solve Problem 1. More in detail, it is necessary to optimize a global-objective function which is a combination of local-objective functions. Upon inspection of problem (13), it becomes evident that it is a specific instance of a more general optimization problem, i.e. a convex optimization problem with separable costs and coupling constraints. This implies that the local objective functions and constraints decompose over the components of the decision vector. In this particular context, approaches that rely on using Lagrangian dual-decomposition and dual methods for solving the problem are of interest, especially because duality yields distributed methods primarily for separable problems [11].

As previously mentioned, although some works exist on distributed algorithms for constraint-coupled optimization problems, some challenges remain, such as primal feasibility recovery and the need for agents to agree on the complete solution vector. To address these issues, this section reviews the distributed optimization method from [27] and applies it to Problem 1. The key contributions of this work include a novel distributed algorithm for constraint-coupled optimization problems that offers local computations, direct computation of primal optimal solution components without averaging, and, notably, privacy preservation through the exchange of only auxiliary variables. As presented in [27], the RSDD method is an approach that utilizes relaxation and successive dual transformations on a constraint-coupled optimization problem. A key advantage of the relaxation approach is its ability to bypass the need for prior verification of local problem feasibility by permitting (scalar) violations of local coupling constraints and enabling distributed implementation.

Through successive applications of the duality principle, the relaxed optimization problem can be solved by applying dual decomposition and subsequently the subgradient method. In the general formulation, the RSDD is applied to a constraint-coupled optimization problem with separable cost function that shares the same structure and assumptions of problem (13). The resulting RSDD algorithm, applied to (13), from the perspective of node i, is based on the following steps

$i)$ calculation of $((g_i^{(k+1)}, \rho_i^{(k+1)}), \boldsymbol{\mu}_i^{(k+1)})$ as a primal-dual optimal solution pair of

$$\begin{aligned} \min_{g_i, \rho_i} \;& f_i(g_i, r_i(t)) + M\rho_i \\ \text{s.t. } & \rho_i \geq 0,\; g_i \in \mathcal{V}_i^l(x_i(t)) \\ h_i^c(g_i, x_i(t), [x]_i(t)) + & \sum_{j\in\mathcal{N}_i} \left(\boldsymbol{\lambda}_{ij}^{(k)} - \boldsymbol{\lambda}_{ji}^{(k)}\right) \leq \rho_i \mathbf{1} \end{aligned} \tag{14}$$

$ii)$ update of the neighboring variables $\boldsymbol{\lambda}_{ij}$ for all $j \in \mathcal{N}_i$

$$\boldsymbol{\lambda}_{ij}^{(k+1)} = \boldsymbol{\lambda}_{ij}^{(k)} - \gamma^k \left(\boldsymbol{\mu}_i^{(k+1)} - \boldsymbol{\mu}_j^{(k+1)}\right) \tag{15}$$

where $k \geq 0$, $\mathbf{1} \triangleq [1, \ldots, 1]^T \in \mathbb{R}^s$ and $\mu_i \in \mathbb{R}^s$, obtained by solving the local problem (14), is the vector of multipliers linked to the inequality constraints in Equation (14). Additionally, the neighboring variables $\boldsymbol{\lambda}_{ij} \in \mathbb{R}^s$, $j \in \mathcal{N}_i$, are the Lagrangian multipliers associated with node i and its set of neighbors $\mathcal{N}_i$. Basically, the algorithm comprises an iterative two-step procedure. Each node $i \in \{1, \ldots, N\}$ maintains a set of variables $((g_i, \rho_i), \boldsymbol{\mu}_i) \in \mathbb{R}^{n_i} \times \mathbb{R} \times \mathbb{R}^s$, representing the primal-dual optimal solution pair of problem (14). Notably, (14) is a localized version of the centralized problem (13), where the coupling with other nodes in the original formulation is replaced by a local term depending solely on neighboring variables $\boldsymbol{\lambda}_{ij}$ and $\boldsymbol{\lambda}_{ji} \in \mathbb{R}^s$, $j \in \mathcal{N}_i$. Moreover, the local version of the coupling constraint is relaxed by allowing a ρ_i positive violation. In addition to minimizing the local f_i, the violation $M\rho_i$, with $M > 0$, is also incorporated into the cost function. At each algorithm iteration, neighboring nodes exchange the updated values of $\boldsymbol{\lambda}_{ij}$. These values are updated according to a suitable linear law that combines neighboring $\boldsymbol{\mu}_i$ values, as defined in (15). More in details, agents can use a step-size denoted by γ^k and initialize the variables $\boldsymbol{\lambda}_{ij}$, $j \in \mathcal{N}_i$, to arbitrary values. In the context of the Command Governor, the optimization problem changes at every time step t, with both the local constraint set and objective function changing due to variations in the initial state $x(t)$ and reference $r(t)$.

Even if the specific objective function and initial state may vary at each time step, the problem structure remains consistent with assumptions like convexity and compactness. When time t is fixed, the cost function of the problem can be expressed as in [27]. Note that $f^{t,\star}$ represents the optimal solution value of the centralized problem at time t, i.e.

$$\begin{aligned} f^{t,\star} &= \|g^\star - r(t)\|_\Psi^2, \\ \text{s.t. } \; & g^\star \in \mathcal{V}(x(t)). \end{aligned} \tag{16}$$

Lemma *1:* **([27, Theorem II.6]):** Let $\boldsymbol{\mu}^\star$ be an optimal solution of the dual problem of problem (14) and $M > 0$ be such that $M > \|\boldsymbol{\mu}^\star\|_\infty$ and the step-size $\gamma^k \geq 0$, for all $k \geq 0$ satisfies the diminishing condition

$$\lim_{k\to\infty} \gamma^k = 0,\; \sum_{k=1}^{\infty} \gamma^k = \infty,\; \sum_{k=1}^{\infty} (\gamma^k)^2 < \infty.$$

Let $\{g_i^{(k)}(t), \rho_i^{(k)}(t)\}_{k\geq 0}$, $i \in \{1, \ldots, N\}$, be a sequence generated by the Distributed Optimization distributed algorithm at time t. Then, the sequence $\{\sum_{i=1}^N \left(f_i(g_i^{(k)}(t)) + M\rho_i^{(k)}(t)\right)\}_{k\geq 0}$ converges to the optimal cost $f^{\star,t}$ of (13).□

Further details can be found in [27]. Basically, if the step-size satisfies the diminishing condition, it can be proved that each local solution obtained using the RSDD algorithm is guaranteed to asymptotically converge to the component of an optimal feasible solution of the original problem. Compared to [27], where the optimal solution for each agent is reached asymptotically, in our approach, agents compute an admissible solution in a finite time within an ϵ_ρ-tightened version of coupling constraints, with $\epsilon_\rho > 0$. From the perspective of the generic node i with neighborhood $\mathcal{N}_i$, a pseudo-code implementing the algorithm is reported below

Algorithm 1: RSDD-based Distributed Optimization
(AGENT i)
INITIALIZE $k = 0$, $\boldsymbol{\lambda}_{ij}^{(0)}$ FOR ALL $j \in \mathcal{N}_i$
OUTPUT $g_i^{(k_f)}$

1: COMPUTE $((g_i^{(k+1)}, \rho_i^{(k+1)}), \boldsymbol{\mu}_i^{(k+1)})$ SOLVING (14) WITH TIGHTENED COUPLING CONSTRAINTS

$$h_i^c(g_i, x_i(t), [x]_i(t)) + \epsilon_\rho \mathbf{1} + \sum_{j\in\mathcal{N}_i} \left(\boldsymbol{\lambda}_{ij}^{(k)} - \boldsymbol{\lambda}_{ji}^{(k)}\right) \leq \rho_i \mathbf{1} \tag{17}$$

2: RECEIVE $\boldsymbol{\mu}_j^{(k+1)}$ AND UPDATE NEIGHBORING VARIABLES USING (15)
3: **IF** $\rho_i^{(k+1)} \leq \epsilon_\rho$ **THEN**
4:     **IF** NEIGHBORING AGENTS CONVERGED **THEN**
5:         SET $k_f = k$, STOP
6:     **ELSE**
7:         SET $k = k + 1$, GO TO 1

Please notice that the number of iterations k at each time t is not fixed. This is due to the fact that the stopping criteria, satisfied by all agents, depends on coupling constraint violations. Also, the condition $\rho_i^{(k+1)} \leq \epsilon_\rho$ is eventually met, since, as proved in [27], $\lim_{k\to\infty} \rho_i^{(k)} = 0$.

Remark 1: Because the RSDD algorithm does not guarantee feasibility for each agent i when the iterations are stopped at any finite step, it could be useful to select constraints as in (17). With this choice, the local coupling constraints penalize violations of the tighter constraints given in (14) (i.e. $h_i^c(g_i) + \sum_{j\in\mathcal{N}_i} \left(\boldsymbol{\lambda}_{ij}^{(k)} - \boldsymbol{\lambda}_{ji}^{(k)}\right) = 0 \Longleftrightarrow h_i^c(g_i) + \sum_{j\in\mathcal{N}_i} \left(\boldsymbol{\lambda}_{ij}^{(k)} - \boldsymbol{\lambda}_{ji}^{(k)}\right) \leq -\epsilon_\rho \mathbf{1}$) and feasibility can be guaranteed. □

The aforementioned formulation enables us to present the Cooperative Distributed Command Governor scheme when executed at each time instant by all agents

Algorithm 2: Cooperative Distributed CG Algorithm (RSDD-CG)
(AGENT i)
INITIALIZE $\boldsymbol{\lambda}_{ij}(0)$ FOR ALL $j \in \mathcal{N}_i$
AT EACH TIME t

1: RECEIVE $\boldsymbol{\lambda}_{ji}(t-1)$, $x_j(t)$ FROM EACH $j \in \mathcal{N}_i$
2: BUILD VECTORS $[\boldsymbol{\lambda}_j]_i(t-1)$ AND $[x]_i(t)$
3: COMPUTE $g_i(t)$, $\boldsymbol{\lambda}_{ij}(t)$ BY MEANS OF **ALGORITHM 1** WITH INPUTS $x_i(t)$, $[x(t-1)]_i$, $[\boldsymbol{\lambda}_j(t-1)]_i$, $\boldsymbol{\lambda}_{ij}(t-1)$
4: APPLY $g_i(t)$
5: TRANSMIT $\boldsymbol{\lambda}_{ij}(t)$ AND $x_i(t)$ TO $\mathcal{N}_i$

Please notice that, since the $\boldsymbol{\lambda}_{ij}$ can be initialized at arbitrary values, in the proposed scheme agents use the values computed at the previous iteration.

V. PROPERTIES

The main properties of the described Supervision scheme are outlined in the following Theorem.

Theorem 1: Consider the Cooperative Distributed CG Algorithm (**Algorithm 2**), let the assumptions of Lemma 1 hold true. Suppose that the problem in **Algorithm 1** is feasible at time $t = 0$, $\rho_i \leq \epsilon_\rho$, $\forall i \in \mathcal{A}$. Then

1) It is feasible for all $t > 0$. The global solution $g(0)$ computed by all agents is an admissible solution of the non-relaxed problem (13);
2) At each time instant $t \in \mathbb{Z}_+$, all agents asymptotically compute their portion of an optimal solution of the original centralized problem $f^{t,\star}$;
3) At each time $t \in \mathbb{Z}_+$, the global action of all agents $i \in \mathcal{A}$ solving **Algorithm 1**, generates an admissible solution that is ϵ_k-close, with $\epsilon_k \in \mathbb{R}$, to the optimal solution of the original centralized problem $f^{t,\star}$.

Proof: Omitted for space reasons. ■

VI. Illustrative Example: Cooperative Connectivity Keeping

This section aims at validating the proposed method through simulation. Experiments were conducted using the CoGoV Toolbox [28], involving two vehicles operating in a 2D space subject to both local and connectivity keeping convex coupling constraints. For this purpose, the dynamics of the i-th vehicle are modeled as

$$\begin{cases} \ddot{x}_i^x(t) = -\dfrac{c}{m}\,\dot{x}_i^x(t) + T_{x,i}(t), \\ \ddot{x}_i^y(t) = -\dfrac{c}{m}\,\dot{x}_i^y(t) + T_{y,i}(t), \end{cases} \tag{18}$$

where $x_i^x(t)$ e $x_i^y(t)$ represent the x and y positions of the vehicle i in a earth-fixed reference frame. The parameter m, chosen to be equal to $10\,[Kg]$, is the mass of the vehicles while $c = 1\,[\frac{Kg}{s}]$ is the friction coefficient. A continuous-time control law has been designed to ensure stability and zero error tracking for constant set-points. The models were discretized using a *Zero-Order Hold* technique with a sampling time of $T_c = 0.1\,[s]$ and have been expressed in the form (3). It is assumed that each agent i is subject to the following local

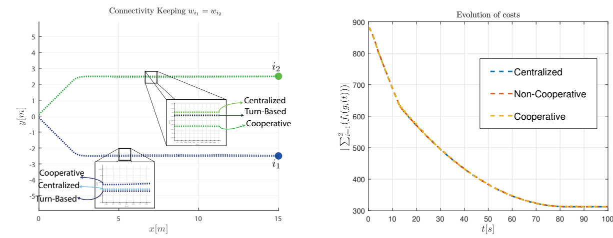
$$\begin{cases} |T_{l,i}(t)| \leq 0.5\,[N], |x_i^l(t)| \leq 20\,[m], \\ \text{with } l = \{x, y\}\,,\ \forall t \in \mathbb{Z}_+, \end{cases} \tag{19}$$

and following coordination coupling constraints

$$\Big\{ ||x_i(t) - x_j(t)||_\infty \leq d_{max},\, \forall j \in \mathcal{N}_i(t), \tag{20}$$

with $d_{max} = 5\,[m]$, $\mathcal{N}_{i_1}(t) = i_2$ and $\mathcal{N}_{i_2}(t) = i_1$, $\forall t \in \mathbb{Z}_+$. The simulating scenario depicted in Figure 1 has been considered, where the two agents, that are located initially close to each other, are instructed to reach two references that violate the coupling constraints. More in detail, the initial positions are $x_{i_1}(0) = [0, -0.1]^T$, $x_{i_2}(0) = [0, 0.1]^T$, while the given references $r_{i_1}(t) = [15, -15]^T$ and $r_{i_2}(t) = [15, 15]^T$, $\forall t \in \mathbb{Z}_+$. Due to this choice, the measured distance between the vehicles grows until they reach the maximum admissible distance allowing connectivity.

For comparison, three CG based supervision methods have been considered, that are the standard centralized CG scheme of [1], the non-cooperative turn-based distributed CG of [10] and the proposed cooperative distributed CG scheme. The *admissibility index* is $\kappa_0 = 20$ while weighting matrices are $\Psi_1 = \Psi_2 = I_2$. Furthermore, the value of $M > 0$ for the proposed approach has been set as $M = 1400$, $\epsilon_\rho = 1e-14$, a step-size sequence, satisfying the diminishing condition assumption, in the form $\gamma^k = \frac{1}{4}k^{-0.955}$ has been used and $\lambda_{i_1 i_2}(0) = \lambda_{i_2 i_1}(0) = 5 \cdot \mathbf{1} \in \mathbb{R}^s$ with $s = 4(\kappa_0 + 1)$.



(a) Trajectories using the three different approaches.

(b) Evolution of the costs.

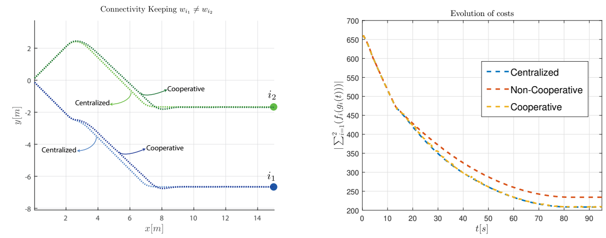
Fig. 1. Trajectories of the vehicles using the centralized approach, the Turn-Based distributed approach and the proposed cooperative distributed approach when $w_{i_1} = w_{i_2} = 1$.

The simulations primarily seek to demonstrate that the cooperative distributed solution closely approximates the centralized solution, whereas the non-cooperative solution does not. To this end, two illustrative case studies are presented. In both cases, we consider different weight assignments for vehicles i_1 and i_2, reflecting varying designer preferences. In the first case, the weights $w_{i_1} = w_{i_2} = 1$ are equal, implying equal priority for both vehicles. Figure 1 compares the trajectories of the vehicles under the three CG schemes. As depicted, all three approaches yield identical behavior and vehicles saturate the connectivity keeping constraints with $x_1^y = -2.5\,[m]$ and $x_2^y = 2.5\,[m]$. However, this is a special case; for any other choice of w_{i_1} and w_{i_2}, the trajectories using non-cooperative approach remain unchanged. More in detail, a designer may desires to prioritize one vehicle over another. For instance, in the second case, where vehicle i_1 has been given higher priority than vehicle i_2 ($w_{i_1} > w_{i_2}$), the centralized solution (which minimizes the combined contribution of both) prioritizes the proximity of vehicle i_1 to its target. In contrast, as previously mentioned, the non-cooperative distributed solution does not consider designer preferences and leads to identical trajectories (Fig. 1(a)) regardless of weight assignments. On the other hand, the cooperative distributed approach consistently yields results closer to the centralized solution (Fig. 2(a)). Furthermore, as illustrated in Fig. 2(b), the non-cooperative solution results in higher total cost compared to the centralized and cooperative distributed approaches. However, it is important to note that the proposed solution requires agents to solve multiple optimization problems and exchange information at each time step t, leading to a significantly higher computational load compared to the non-cooperative distributed approach.

In Figure 3 we show the measured distance between the two vehicles at each iteration t, both using the Centralized 3(a) and the proposed 3(b) approaches. It is interesting to notice that in the latter case the measured distance never violates the connectivity keeping constraints.

VII. Conclusions

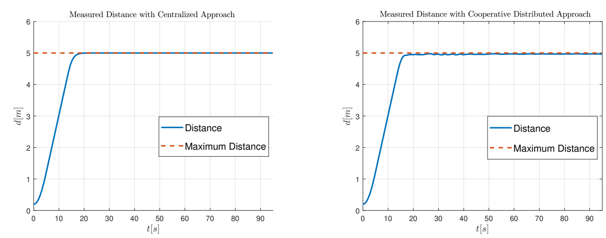
This paper presented a novel cooperative distributed CG



(a) Trajectories using the centralized and the proposed approach.

(b) Evolution of the costs.

Fig. 2. Trajectories of the vehicles using the centralized approach, the Turn-Based distributed approach and the proposed cooperative distributed approach when $w_{i_1} = 1$ and $w_{i_2} = 0.5$.



(a) Evolution of the distance between the two vehicles with the centralized approach.

(b) Evolution of the distance between the two vehicles using the proposed approach.

Fig. 3. Evolution in time of the distance between vehicle i_1 and vehicle i_2 with the centralized approach and with the proposed cooperative distributed approach when $w_{i_1} = 1$ and $w_{i_2} = 0.5$.

scheme for supervising dynamically coupled interconnected linear systems subject to local and coupling constraints. The proposed scheme, based on the RSDD algorithm, addressed constrained coordination problems while ensuring privacy by avoiding the exchange of information related to objective functions, constraints, or admissible commands. An illustrative example of two vehicles operating in a 2D space, subject to connectivity constraints, validated the effectiveness of the proposed approach. Future research endeavors could explore extending the results to Command Governor strategies for non-convex constraints based on mixed integer optimization.

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